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Shielding from Space Radiations

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ABSTRACT

This Progress Report covering the period of June 1, 1994 to December 1, 1994 presents the development of an analytical solution to the heavy ion transport equation in terms of a two-layer Green's function formalism. The mathematical developments are recasted into an efficient computer code for space applications. The efficency of this algorithm is accomplished by a nonperturbative technique of extending the Green's function over the solution domain. The code may also be applied to accelerator boundary conditions to allow code validation in laboratory experiments. Correltion for the isotopic version of the code with 80 isotopes present for a two layers target material in water with experiment is good. A number of computationally efficient interpolation algorithms have replaced the previous versions in the code, with an increase of 20% in overall code efficency. Work is under way to further enhance the efficency of the code.

INTRODUCTION

Future NASA missions will be limited by exposure to space radiations unless adequate shielding is provided to protect men and equipments from such radiations. Adequate methods required to estimate the damage caused by such radiations behind various shields can be evaluated prior to commitment to such missions.

From the inception of the Langley Research Center heavy ion (HZE) shielding program (refs. 2-4), there has been a continued, close relationship between code development and laboratory experiment (ref. 4). Indeed, the current research goal is to provide computationally efficient high charge and energy ion (HZE) transport codes which can be validated with laboratory experiments and subsquently applied to space engineering design. In practice, two streams of code development have prevailed due to the strong energy dependence of necessary atomic/molecular cross sections and the near singular nature of the laboratory beam boundary conditions (refs. 5-7). The atomic/molecular cross section dependence is adequately dealt with by using the methods of Wilson and Lamkin (ref. 8), allowing efficient numerical procedures to be developed for space radiations (refs. 7,9-11). Although these codes could conceivably be applied to the laboratory validation, methods to control truncation and discretization errors would bear little resemblance to the space radiation codes attempting to be validated. Clearly, a radical reorientation is required to achieve the validation goals of the current NASA space radiation shielding program, and such an approach in terms of a multi-layered Green's function formalism as an extension to a previously developed single-layered algorithm is the main thrust of this continuing research and is briefly described below.

GREEN'S FUNCTION METHODS

Transport Equations and Conventional Approach

The transport equation for high energy heavy ions is usually simplified by assuming the straightahead approximation and neglecting the target secondary fragments (ref. 3 et. al) and is written as

$$\left[\frac{\partial}{\partial x} - \frac{\partial}{\partial E} \, \tilde{S}_j(E) + \sigma_j \right] \phi_j(x, E) = \sum_k \sigma_{jk} \, \phi_k(x, E) \tag{1}$$

where $\phi_j(x, E)$ is the ion flux at x with energy E (MeV/amu), $\tilde{S}_j(E)$ is the change in E per unit distance, σ_j the total macroscopic absorption cross section and σ_{jk} the macroscopic cross section for collision of ion type k to produce an ion of type j. The solution to equation (1) is to be found subject to the boundary condition

$$\phi_j(0, E) = f_j(E) \tag{2}$$

which for laboratory beams has only one value of j for which $f_j(E)$ is not zero and that $f_j(E)$ is described by a mean energy spread σ such that

$$f_j(E) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-(E - E_o)^2 / 2\,\sigma^2 \right] \tag{3}$$

The solution to equation (1) is given by superposition of Greens function G_{jk} as

$$\phi_j(x, E) = \sum_k \int G_{jk}(x, E, E') \, f_k(E') dE' \tag{4}$$

where Green function is a solution of

$$\left[\frac{\partial}{\partial x} - \frac{\partial}{\partial E} \, \tilde{S}_j(E) + \sigma_j \right] G_{jm}(x, E, E_o) = \sum_k \sigma_{jk} G_{km}(x, E, E_o) \tag{5}$$

subject to the boundary condition

$$G_{jm}(0, E, E_o) = \delta_{jm} \delta(E - E_o)$$

(6)

The above equations can be simplified by transforming the energy into the residual range as

$$r_j = \int_0^E dE'/\tilde{S}_j(E')$$

(7)

and defining new field functions as

$$\psi_j(x, r_j) = \tilde{S}_j(E)\phi_j(x, E)$$

(8)

$$G_{jm}(x, r_j, r'_m) = \tilde{S}_j(E) \, G_{jm}(x, E, E')$$

(9)

$$\hat{f}_j(r_j) = \tilde{S}_j(E) \, f_j(E)$$

(10)

equation (5) becomes

$$\left| \frac{\partial}{\partial x} - \frac{\partial}{\partial r_j} + \sigma_j \right| G_{jm}(x, r_j, r'_m) = \sum_k \frac{r_j}{r_k} \sigma_{jk} \, G_{km}(x, r_k, r'_m)$$

(11)

with the boundary condition

$$G'_{jm}(0, r_j, r'_m) = \delta_{jm} \delta(r_j - r'_m)$$

(12)

and with the solution to the ion fields given by

$$\psi_j(x, r_j) = \sum_m \int_0^\infty G_{jm}(x, r_j, r'_m) \hat{f}_m(r'_m) dr'_m$$

(13)

Note ν_j is the range scale factor as $\nu_j r_j = \nu_m r_m$ and is taken as $\nu_j = Z_j^2/A_j$. The solution to equation (11) is written as a perturbation series

$$U_{jm}(x, r_j, r'_m) = \sum_i U_{jm}^{(i)}(x, r_j, r'_m) \tag{14}$$

where

$$U_{jm}^{(o)}(x, r_j, r'_m) = g(j)\, \delta_{jm}\, \delta(x + r_j - r'_m) \tag{15}$$

and

$$U_{jm}^{(1)}(x, r_j, r'_m) \approx \frac{\nu_j \sigma_{jm} g(j, m)}{x(\nu_m - \nu_j)} \tag{16}$$

where $U_{jm}^{(1)}(x, r_j, r'_m)$ is zero unless

$$\frac{\nu_j}{\nu_m}(r_j + x) \le r'_m \le \frac{\nu_j}{\nu_m} r_j + x \tag{17}$$

for $\nu_m > \nu_j$. If $\nu_j > \nu_m$ as can happen in neutron removal, the negative of equation (16) is used and upper and lower limits of equation (17) are switched. The higher order terms are approximated as

$$U_{jm}^{(i)}(x, r_j, r'_m) \approx \sum_{k_1, k_2, \ldots k_{i-1}} \frac{\nu_j \sigma_{jk_1} \sigma_{k_1 k_2} \cdots \sigma_{k_{i-1} m} g(j, k_1, k_2, \cdots, k_{i-1}, m)}{x(\nu_m - \nu_j)} \tag{18}$$

In the above

$$g(j) = e^{-\sigma_j x} \tag{19}$$

and

and

$$g(j_1, j_2, \ldots, j_n, j_{n+1}) = \frac{g(j_1, j_2, \ldots, j_{n-1}, j_n) - g(j_1, j_2, \ldots, j_{n-1}, j_{n+1})}{\sigma_{j_{n+1}} - \sigma_{j_n}} \qquad (20)$$

Note that $G_{jm}^{(i)}(x, r_j, r_m')$ is purely dependent on x for $i > 0$, which we represent as $G_{jm}^{(i)}(x)$. (See ref. 3.) In terms of the above, the solution to equation (1) becomes (from ref. 3)

$$\psi_j(x, r_j) = e^{-\sigma_j x} \hat{f}_j(r_j + x)$$

$$+ \sum_{m,i} G_{jm}^{(i)}(x) \left[\widehat{F}_m(r_{m\ell}') - \widehat{F}_m(r_{mu}') \right] \qquad (21)$$

In equation (21), r_{mu}' and $r_{m\ell}'$ are given by the upper and lower limits of the inequality of equation (17). The symbol $\widehat{F}_m(r_m')$ refers to the integral spectrum

$$\widehat{F}_m(r_m') = \int_{r_m'}^{\infty} \hat{f}_m(r)\, dr$$

We note that

$$\widehat{F}_m(r_m') \equiv F_m(E')$$

with

$$F_m(E') = \int_{E'}^{\infty} f_m(E)\, dE \qquad (22)$$

and

$$r_m' = \int_0^{E'} dE / \tilde{S}_m(E)$$

We now introduce nonperturbative terms for the summation in equation (21).

. First, we recall that the g function of n arguments was generated by the perturbation solution of the transport equation neglecting ionization energy loss (ref. 1) given by

$$\left[\frac{\partial}{\partial x} + \sigma_j\right] g_{jm}(x) = \sum_k \sigma_{jk} \, g_{km}(x) \tag{23}$$

subject to the boundary condition

$$g_{jm}(0) = \delta_{jm} \tag{24}$$

for which the solution is

$$g_{jm}(x) = \delta_{jm} g(m) + \sigma_{jm} g(j, m) + \cdots \tag{25}$$

It is also true that

$$g_{jm}(x) = \sum_k g_{jk}(x - y) g_{km}(y) \tag{26}$$

for any positive values of x and y. Equation (26) may be used to propagate the function $g_{jm}(x)$ over the solution space from very thin shield solutions. Equation (14) is then rewritten as

$$G_{jm}(x, r_j, r'_m) \approx e^{-\sigma_j x} \, \delta_{jm} \, \delta(x + r_j - r'_m) + \nu_j \left[g_{jm}(x) - e^{-\sigma_j x} \delta_{jm} \right] / [x(\nu_m - \nu_j)] \tag{27}$$

and the approximate solution of equation (1) is given by

$$\psi_j(x, r_j) = e^{-\sigma_j x} \, f(r_j + x)$$
$$+ \sum_m \frac{\nu_j \left[g_{jm}(x) - e^{-\sigma_j x} \, \delta_{jm} \right]}{x(\nu_m - \nu_j)} \left| F_m(r'_{mu}) - F_m(r'_{m\ell}) \right| \tag{28}$$

Green's Function Methods in a Shielded Medium

The major simplification in the Green's function method results from

the fact that the scaled spectral distribution of secondary ions to a first approximation depends only on the depth of target penetration as seen in equations (16), (18), and (27). The first approach to a multi-layered Green's function will rely on this observation and assume its validity for multi-layered shields.

Consider a domain labeled as 1 which is shielded by a second domain labeled as 2, the number of type j ions at depth x in 1 due to type m ions incident on domain 2 of thickness y is

$$g_{12jm}(x,y) = \sum_k g_{1jk}(x)\, g_{2km}(y)$$

(29)

The leading term in equation (29) is the penetrating primaries as

$$g_{12jm}(x,y) = e^{-\sigma_{1j}x - \sigma_{2j}y}\,\delta_{jm} + [g_{12jm}(x,y) - e^{-\sigma_{1j}x - \sigma_{2j}y}\,\delta_{jm}]$$

(30)

where all higher order terms are in the bracket of equation (30).

The first term of scaled Green's function is then

$$G^{(o)}_{12jm}(x,y,r_j,r'_m) = e^{-\sigma_{1j}x - \sigma_{2j}y}\delta_{jm}\,\delta[x + r_j - (r'_m - \rho y)]$$

(31)

where ρ is the range scale factor for the two media

$$\rho = R_{1j}(E)/R_{2j}(E)$$

(32)

A sigle value for ρ is taken corresponding to 600 MeV/amu. The secondary contribution is similarly found by noting the equation (17) becomes

$$\frac{\nu_j}{r_m'}(r_j + x + \rho y) \le r_m' \le \frac{\nu_j}{\nu_m} r_j + x + \rho y \tag{33}$$

from which the average spectrum is evaluated. The full approximate Green's function is then

$$G_{12jm}(x, y, r_j, r_m') \approx e^{-\sigma_{1j}x - \sigma_{2j}y} \delta_{jm}\delta(x + \rho y + r_j - r_m')$$

$$+ \nu_j \left| g_{12jm}(x, y) - e^{-\sigma_{1j}x - \sigma_{2j}y}\delta_{jm} \right| / |(x + \rho y)(\nu_m - \nu_j)| \tag{34}$$

Equation (34) is the first approximation to the Green's function in a shielded medium (two layers) and can be modified to multiple layers. The first spectral modification is considered next. It is easy to show that the first collision term has the properties

$$G_{12jm}^{(1)}(x, y, r_j, r_m') = \begin{cases} \dfrac{\nu_j \sigma_{1jm} e^{-\sigma_{1m}x - \sigma_{2m}y}}{|\nu_m - \nu_j|} & \text{for } r_m' = r_{mu}' \\[3mm] \dfrac{\nu_j \sigma_{2jm} e^{-\sigma_{1j}x - \sigma_{2j}y}}{|\nu_m - \nu_j|} & \text{for } r_m' = r_{m\ell}' \end{cases} \tag{35}$$

These properties are used to correct the average spectrum as

$$G_{12jm}^{(1)}(x, y, r_j, r_m') = \frac{\nu_j\, g_{12jm}^{(1)}(x, y)}{|\nu_m - \nu_j|(x + \rho y)} + b_{jm}(x, y)(r_m' - \bar{r}_m) \tag{36}$$

where $g_{12jm}^{(1)}(x, y)$ is the first collision term of equation (34) and

$$\bar{r}_m = (r_{mu}' + r_{m\ell}')/2 \tag{37}$$

is the midpoint of $\bar{r}_m$ between its limits given by equation (33). The term of equation (36) has the property that

$$\int_{r_{m\ell}'}^{r_{mu}'} b_{jm}(x, y)(r' - \bar{r}_m) dr' = 0 \tag{38}$$

ensuring that the first term of equation (36) is indeed the average spectrum as required. The spectral slope parameter is found to be

$$b_{jm}(x,v) = v_j v_m (\sigma_{1jm} e^{-\sigma_{1m}x - \sigma_{2m}v} - \sigma_{2jm} e^{-\sigma_{1j}x - \sigma_{2j}v}) / \left[(x + \rho v)(v_m - v_j)(v_m - v_j) \right] \tag{39}$$

A similarly simple spectral correction could be made to the higher order terms. The spectral correction given in equation (39) is included in the present Green's function code.

Solution for Laboratory Beams

Using the boundary condition appropriate for laboratory beams given by equation (3), cumulative spectrum given by

$$F_j(E) = \frac{1}{2} \left[1 - erf \left(\frac{E - E_0}{\sqrt{2}\,\sigma} \right) \right] \tag{40}$$

and the cumulative energy moment needed to evaluate the spectral correction given by

$$\bar{E}_j(E) = \frac{1}{2} E_0 \left[1 - erf \left(\frac{E - E_0}{\sqrt{2}\,\sigma} \right) \right] + \frac{\sigma}{\sqrt{2\pi}} \exp \left[-\frac{(E - E_0)^2}{2\sigma^2} \right] \tag{41}$$

The average energy on any subinterval (E_1, E_2) is then

$$\bar{E} = \left[\bar{E}_j(E_1) - \bar{E}_j(E_2) \right] / \left[F_j(E_1) - F_j(E_2) \right] \tag{42}$$

and the beam generated flux becomes

$$\psi_j(x, y, r_j) = e^{-\sigma_{1j}x - \sigma_{2j}y}\, \hat{j}_j(r_j + x + \rho y) \tag{43}$$

$$+ \sum_{m,\ell} v_{jm}^{(i)}(x, y) \left| F_m(r'_{mu}) - F_m(r'_{m\ell}) \right|$$

$$+ \sum_m b_{jm}^{(i)}(x, y)\, \left| r'_m(\bar{E}) - F'_m \right| \left| F_m(r'_{mu}) - F_m(r'_{m\ell}) \right|$$

where $\bar{E}$ is evaluated using equation (42) with E_1, and E_2 as the lower and upper limit asscciated with $r'_{m\ell}$ and r'_{mu} .

DISCUSSION OF RESULTS

An algorithm based on the one-layered Green's function formalism was developed and tested during the previous phase of this research project. The current version of this code is computationally efficient for all terms. In order to implement the formulation presented in the theory section of the current proposal for multi-layered targets, the following tasks were undertaken:

1. A direct conversion from formulation to coding based on the described procedures was performed. This resulted in a first generation two-layered algorithm which is still computationally inefficient, especially for higher order terms, and requires further work. A 20% computational efficency was achieved by applying efficient interpolation routines to the developing code.

2. The resulting algorithm was tested against the results of a recent two-layered experiments for correlation purposes and code efficency studies.

3. The tested version of the two-layered algorithm was used as a mean of evaluating the shielding properties of different two-layered target materials as they are exposed to galactic cosmic radiations (GCR).

Furthermore two statistical (Monte Carlo) transport codes were used to carry out the following tasks:

3. Using a low to medium energy Monte Carlo transport code named TIGER, one and three dimensional energy deposition for electrons and

photons in various target materials with and without an Aluminum shielding was calculated. The results was used to study the depth dose relations in geometries of interest to radiation shielding.

4. The capabilities of a high energy Monte Carlo transport code named EGS4 was explored. EGS4 is a transport code for electrons, positrons, and photons, and has the flexibility of user interfacing through MORTRAN3 language. With this capability decay of zero pions can be simulated.

REFERENCES

1. J. W. Wilson, R. C. Costen, J. L. Shinn and F. F. Badavi, "Green's Function Method in Heavy Ion Shielding," NASA TP-3311, March 1993.

2. J. W. Wilson and C. M. Costner, "Nucleon and Heavy Ion Total and Absorption Cross Section for Selected Nuclei," NASA TN D-8107, December 1975.

3. J. W. Wilson, "Analysis of the Theory of High Energy Ion Transport," NASA TN D-8381, March 1977.

4. J. W. Wilson, "Depth Dose Relations for Heavy Ion Beams," Va. J. of Sci. 28, 136-138 (1977).

5. J. W. Wilson, "Heavy Ion Transport in the Straightahead Approximation," NASA TP-2178, June 1983.

6. J. W. Wilson, L. W. Townsend, H. B. Bidasaria, W. Schimmerling, M. Wong and J. Howard, " Ne Depth Dose Relations in Water, "Health Physics 46, 1101 (1984).

7. J. W. Wilson and F. F. Badavi, "Methods of Galactic Heavy Ion Transport," Radiat. Res. 108, 231 (1986).

8. J. W. Wilson and S. L. Lamkin, "Perturbation Approximation for Charged Particle Transport in One Dimension," Nucl. Sci. and Eng. 57, 292-299 (1975).

9. J. W. Wilson, L. W. Townsend, S. Y. Chun, S. L. Lamkin, B. D. Ganapol, B. S. Hong, W. W. Buck, F. Khan, F. A. Cucinotta and J. E. Nealy, "BRYNTRN: A Baryon Transport Model, " NASA TP-2887, 1989.

10. J. L. Shinn, J. W. Wilson, M. Weyland and F. A. Cucinotta, "Improve-
    ments in the computational Accuracy of BRYNTRN (A Baryon Transport
    Code," NASA TP-3093, 1991.

11. J. L. Shinn and J. W. Wilson, "An Efficient HZETRN (A Galactic Cosmic
    Ray Transport Code)," NASA TP-3147, 1991.

12. J. L. Shinn, J. E. Nealy, L. W. Townsend, W. Lawrence, and J. W.
    Wilson, "Galactic Cosmic Ray Radiation Levels in Spacecraft in
    Interplanetary Missions," Invited Paper to the COSPARXXIX Planetary
    meeting, August 1992.

13. J. W. Wilson, F. F. Badavi, R. C. Costen, and J. L. Shinn, "Non-
    perturbative Methods in HZE Transport," NASA TP-3363.